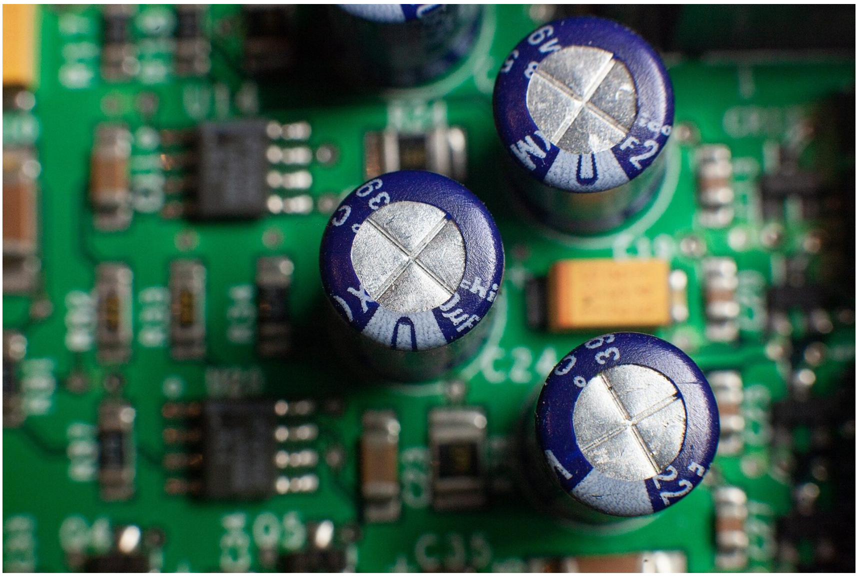
Capacitance

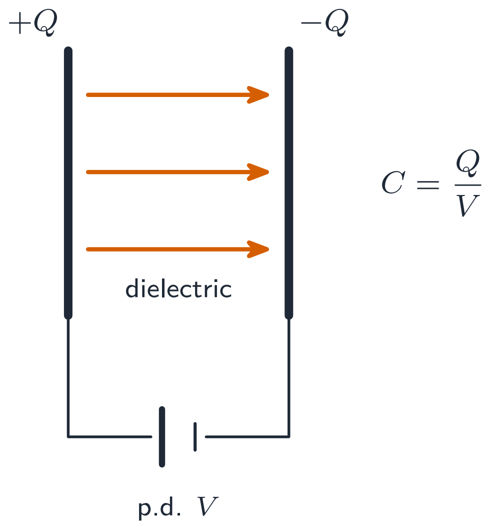
A-Level Physics

Capacitance



Capacitors store electric charge in a circuit.

A **capacitor** 电容器 stores charge. The simplest one is two parallel **conductor** 导体 plates with an **insulator** 绝缘体 (a **dielectric** 电介质, or just **vacuum** 真空 / air) between them. Connected to a battery, charge $+Q$ builds up on one plate and $-Q$ on the other, with a **potential difference** 电势差 V across the gap.



A parallel-plate capacitor: equal and opposite charges on two plates with a p.d. V across the gap

The **capacitance** 电容 C of any capacitor (or any isolated conductor) is

$$C = \frac{Q}{V}.$$

This applies to:

- an **isolated sphere** holding charge Q at potential V (zero at infinity). For radius r, $V = Q/(4\pi\varepsilon_0 r)$, so $C = 4\pi\varepsilon_0 r$.
- a **parallel-plate capacitor**: charges $\pm Q$ on the plates, p.d. V between them.

Unit: **farad** 法拉 (F) $=$ C V^{-1}. A farad is huge, so real capacitors run from pF to mF.

Capacitance is **constant** for a given capacitor (set by its size and dielectric). Doubling the charge doubles the voltage, so $C = Q/V$ stays the same.

Worked example. A 100 μF capacitor is charged to 12 V. Find the charge stored.

$$Q = CV = (100 \times 10^{-6})(12) = 1.2 \times 10^{-3}\ \text{C}\ (= 1.2\ \text{mC}).$$

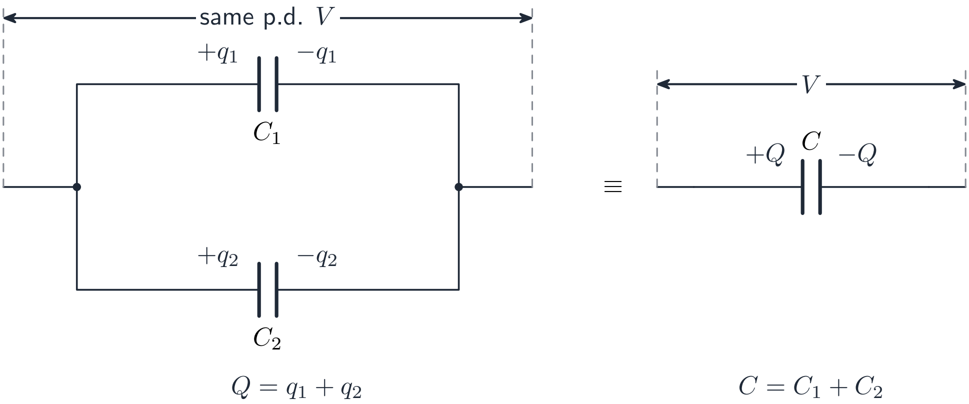
Real capacitors range from large electrolytic cans (high capacitance) down to tiny film and ceramic types – from pF up to mF

Combining capacitors

Capacitors in parallel share the same p.d. V. The total charge is the sum:

$$Q_{\text{total}} = C_1V + C_2V + \dots, \qquad \text{so} \qquad C_{\text{parallel}} = C_1 + C_2 + \dots$$

A parallel combination has **larger** capacitance than any one capacitor.

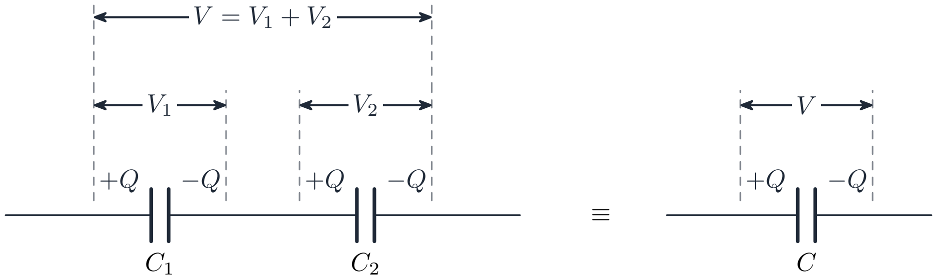


Capacitors in parallel share the same p.d.; the charges add

Capacitors in series carry the **same charge** Q. The total p.d. is the sum:

$$V_{\text{total}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \dots, \qquad \text{so} \qquad \frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

A series combination has **smaller** capacitance than any one capacitor.



Capacitors in series carry the same charge; the p.d.s add

Note: these rules are the **opposite** of those for resistors (resistors sum in series; capacitors sum in parallel), because $C = Q/V$ has V on the bottom while $R = V/I$ has I on the bottom.

Energy stored in a capacitor

Charging a capacitor from 0 to Q needs work, because each extra bit of charge is pushed against the p.d. already there. When the charge is q, the p.d. is $V(q) = q/C$, so adding a small charge dq needs work $V\,dq$. The total work is

$$W = \int_0^Q \frac{q}{C}\,dq = \frac{Q^2}{2C}.$$

Using $V = Q/C$, this is the **energy 能量** stored:

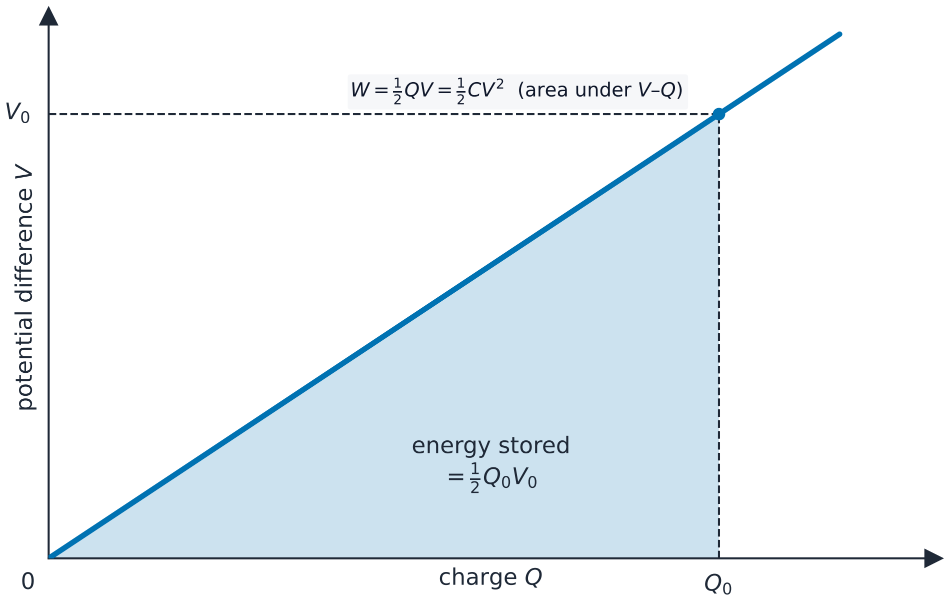
$$W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}.$$

Worked example. Find the energy stored in a 100 μF capacitor charged to 12 V.

$$W = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(100 \times 10^{-6})(12)^2 = 7.2 \times 10^{-3}\ \text{J}\ (= 7.2\ \text{mJ}).$$

Reading the Q–V graph

A plot of V against Q is a straight line through the origin with gradient $1/C$. The energy stored is the **area** under the line up to a given charge Q, which is the triangle $\frac{1}{2}QV$. The factor $\frac{1}{2}$ is there because the average p.d. during charging is $V/2$ (it grows from zero to V), not V.



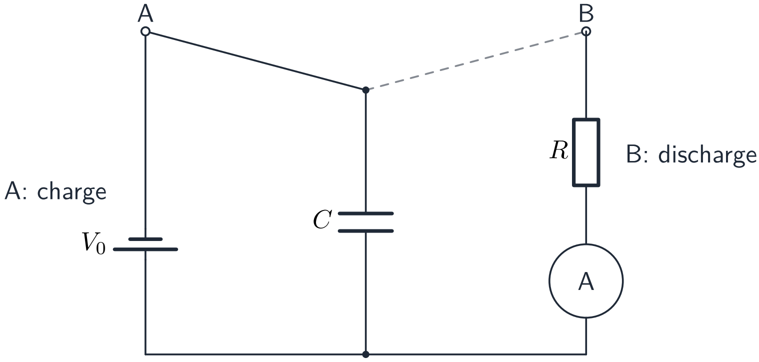
The energy stored is the area under the potential–charge line (the triangle $\frac{1}{2}QV$)

Why charging is "half efficient"

Connect a capacitor C to an ideal battery of e.m.f. V through a wire. The capacitor stores $\frac{1}{2}CV^2$, but the battery supplies charge $Q = CV$ at e.m.f. V, giving out $QV = CV^2$. The other half is lost as heat in the wire — whatever the wire's resistance.

Capacitor discharging through a resistor

A capacitor C charged to V_0 is connected through a switch to a **resistor** 电阻器 of **resistance** 电阻 R. When the switch closes at $t = 0$, the capacitor discharges.



A capacitor charges through switch A, then discharges through the resistor via switch B

Setting up the equation

By **Kirchhoff's second law** 基尔霍夫第二定律 around the loop, $V_C = V_R$. Using $V_C = Q/C$, $V_R = IR$ and $I = -dQ/dt$:

$$\frac{Q}{C} = -R\frac{dQ}{dt}.$$

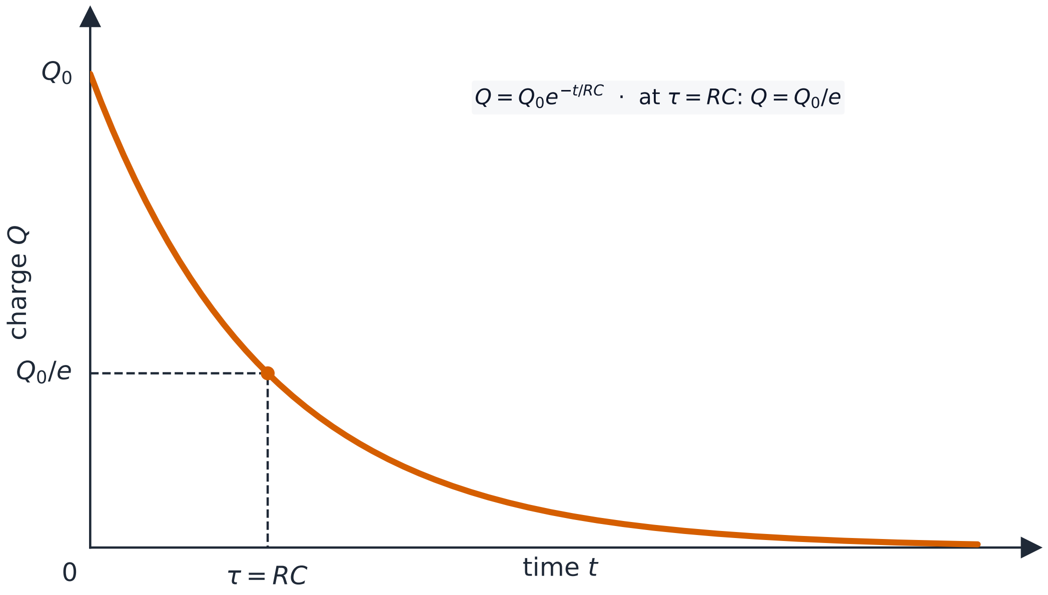
This is solved by an **exponential decay** 指数衰减 with time constant RC.

Discharge equations

Charge Q, p.d. V and current I all decay exponentially with the same time constant:

$$Q = Q_0 e^{-t/(RC)}, \qquad V = V_0 e^{-t/(RC)}, \qquad I = I_0 e^{-t/(RC)},$$

with $I_0 = V_0/R$.



Charge decays exponentially during discharge, falling to Q_0/e after one time constant RC

Time constant

$$\tau = RC$$

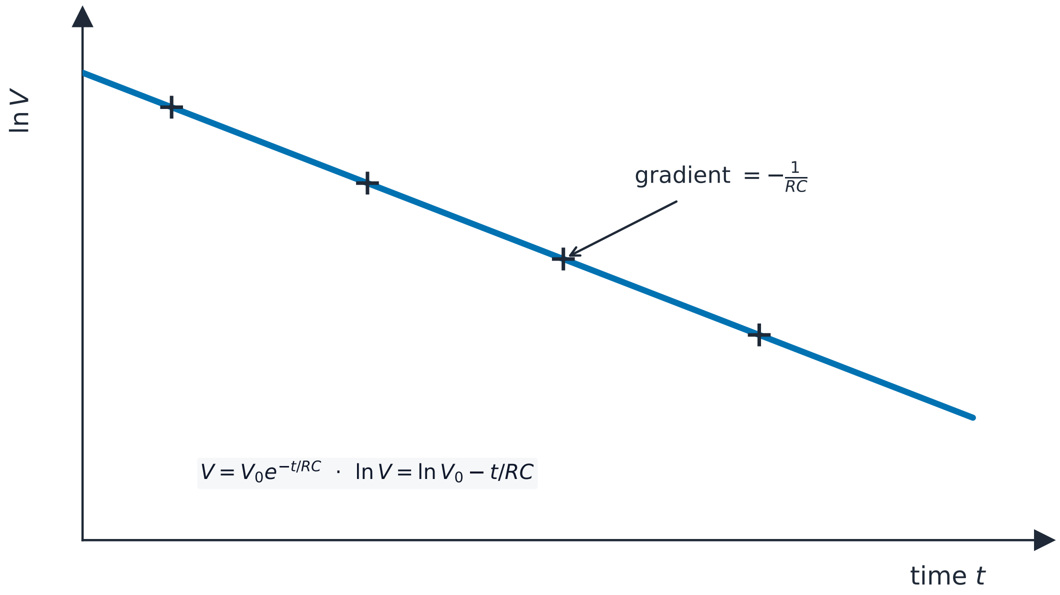
is the **time constant** 时间常数 (in seconds: $\Omega \cdot \mathrm{F} = \mathrm{s}$). It is the time for a decaying quantity to fall to $1/e \approx 0.37$ (about 37%) of its starting value. After 2τ it is at about 13.5%; after 5τ, below 1%.

Worked example. A 100 μF capacitor charged to 12 V is discharged through a 47 kΩ resistor. Find the time constant and the voltage after one time constant.

$$\tau = RC = (47 \times 10^3)(100 \times 10^{-6}) = 4.7 \text{ s}.$$

After one time constant the voltage falls to $1/e$ of its start: $V = 12 \times 0.37 \approx 4.4$ V.

To find τ from a curve: read the time to fall to $1/e$ of the start. Or take logs: $\ln(V/V_0) = -t/(RC)$, so a plot of $\ln V$ against t is a straight line with gradient $-1/(RC)$.



A graph of $\ln V$ *against time is a straight line of gradient* $-1/(RC)$

Reading graphs during discharge

- Q against V_C: since $Q = CV$ always, this is a straight line through the origin with gradient C. Discharge moves the point from (V_0, Q_0) down to $(0, 0)$.
- I against V_C: since $I = V_C/R$, this is a straight line through the origin with gradient $1/R$, so you can find R.

Common exam questions

Given a discharge curve $V(t)$ or $Q(t)$:

- read the start value V_0 or Q_0 at $t = 0$.
- read the time to fall to $V_0/e \rightarrow$ time constant $\tau = RC$.
- given R, find $C = \tau/R$ (or the other way round).
- predict a later value with the exponential formula.

Exam tips

- $C = Q/V$; energy stored $W = \frac{1}{2}QV = \frac{1}{2}CV^2$ (the $\frac{1}{2}$ is the area under the Q-V graph).
- Combine capacitors the **opposite way to resistors** (parallel add, series reciprocal).
- Discharge is exponential: $Q = Q_0 e^{-t/RC}$; the time constant $\tau = RC$ is when the charge falls to 37%.