Gravitational fields

A-Level Physics

Gravitational fields

Definition

A **gravitational field** 重力场 is a region where a **mass** 质量 feels a **force** 力 from other masses. The **gravitational field strength** 重力场强度 g at a point is the **gravitational force per unit mass** on a small **test mass** 检验质量 placed there:

$$g = \frac{F}{m}.$$

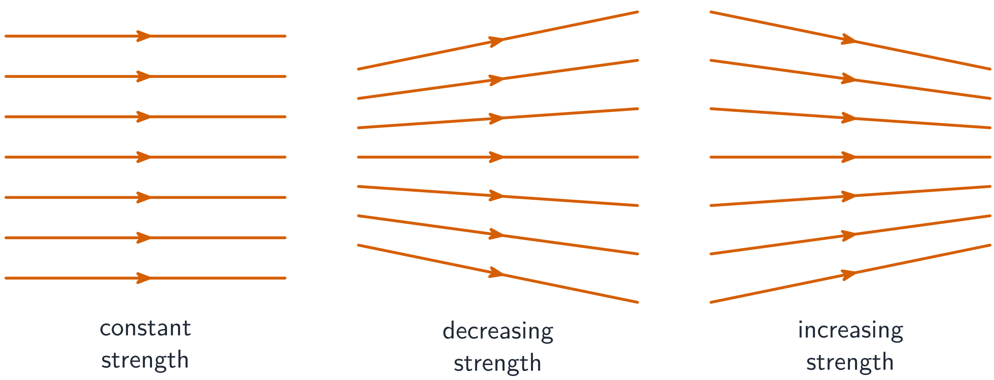
Unit: N kg^{-1} (the same as m s^{-2} — the acceleration of free fall in the field). g is a **vector** 矢量, pointing the way the force acts — towards the source mass.

Field lines

A gravitational field is drawn with **field lines** 场线 that point the way the force acts on a test mass:

- around a **point mass** 质点 or a uniform sphere (treated as a point mass from outside), the field lines are **radial** 径向, pointing **inwards**.
- near the Earth's surface over a small area, the field lines are nearly **parallel and equally spaced**, pointing straight down — a **uniform field** 匀强场.

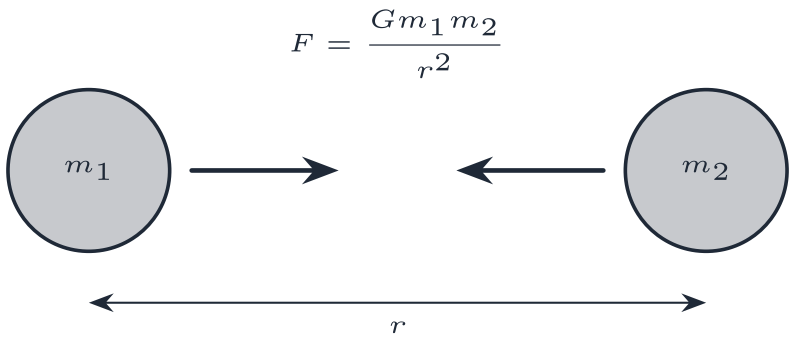
Closer lines mean a stronger field.



Field-line spacing shows the field strength — closer lines mean a stronger field

Newton's law of gravitation

For two point masses m_1, m_2 a distance r apart, the force on each is



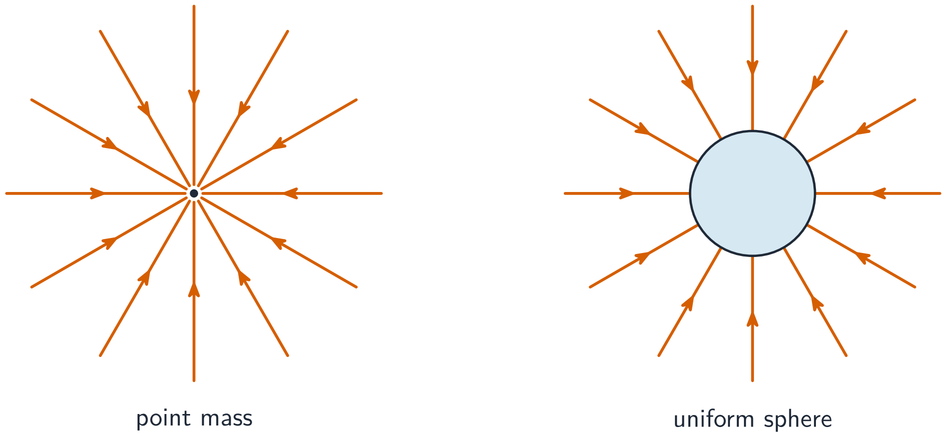
Two masses attract along the line joining them

$$F = \frac{Gm_1m_2}{r^2},$$

pulling them together along the line joining them. This is **Newton's law of gravitation 万有引力定律**. The constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} is the **universal gravitational constant 万有引力常量**.

Spheres treated as point masses

For a uniform sphere (such as a planet or star), the field at any point **outside** is the same as that of a point mass equal to the total mass at the centre. So from above the surface, you can treat the Earth as a point mass at its centre. (Points inside a sphere are different, and are not in the syllabus.)



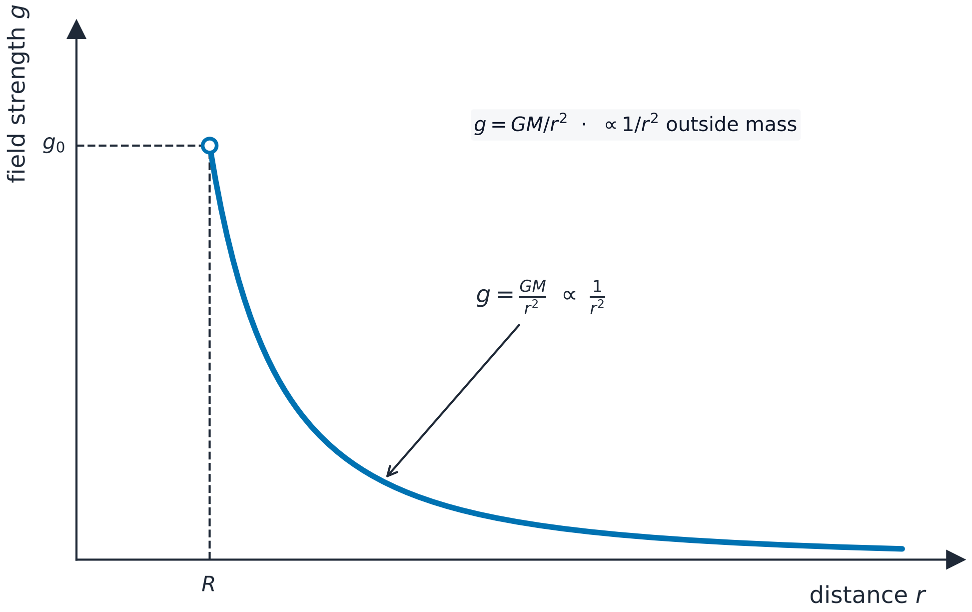
Outside a uniform sphere the field is radial, exactly like a point mass at the centre

Field strength from a point mass

Put the gravitational force on a test mass m at distance r from a point mass M into $g = F/m$:

$$F = \frac{GMm}{r^2}, \qquad g = \frac{GM}{r^2}.$$

So g falls off as $1/r^2$ as you move away from the source.



Field strength falls off as $1/r^2$ with distance from a point mass

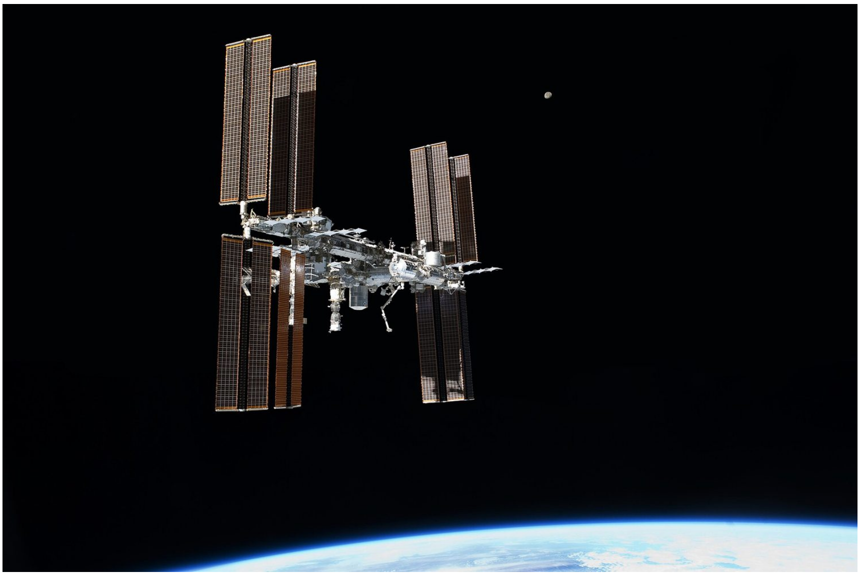
Worked example. Find the gravitational field strength at the Earth's surface. (Earth's mass $M = 6.0 \times 10^{24}$ kg, radius $R = 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.)

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{(6.4 \times 10^6)^2} \approx 9.8 \text{ N kg}^{-1}.$$

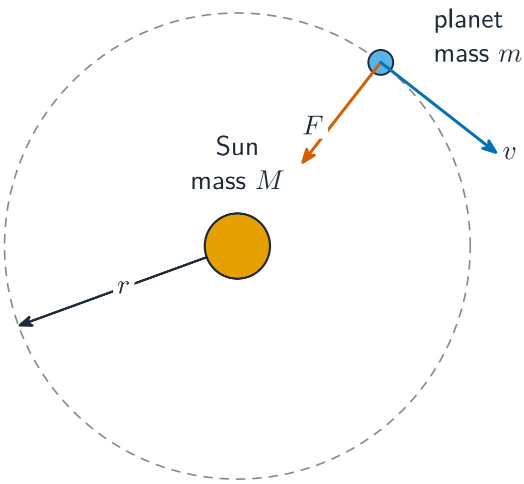
Why g is nearly constant near the Earth's surface

The Earth's radius is $R \approx 6.4 \times 10^6$ m. Rising to height h changes the distance from the centre from R to $R + h$. For $h \ll R$ (any building or mountain), $(R + h)/R \approx 1$, so g barely changes — going from 5 m to 10 m high changes r by about one part in a million. In the laboratory, g is effectively constant.

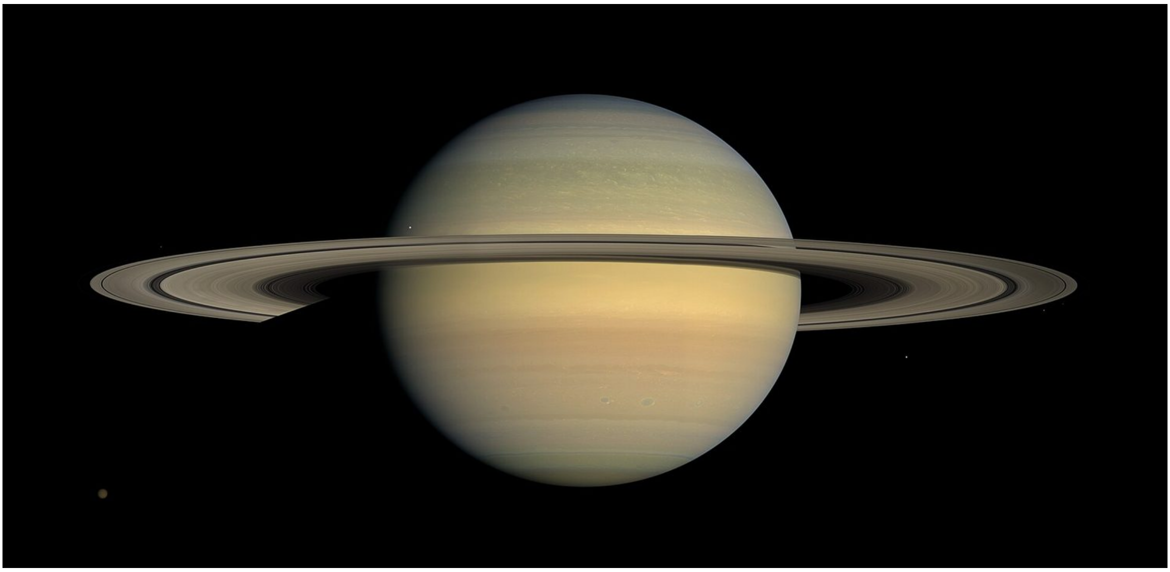
Orbital motion in a gravitational field



The International Space Station orbits Earth, held in its path by gravity.



Gravity provides the centripetal force that keeps a planet in a circular orbit



Saturn, its rings (countless small orbiting pieces) and its moons are all held in orbit by gravity

For a **satellite** 卫星 of mass m in a circular **orbit** 轨道 of radius r around a body of mass M, gravity provides the **centripetal force** 向心力:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}.$$

Cancel m (the orbital speed does not depend on the satellite's mass):

$$v = \sqrt{\frac{GM}{r}}.$$

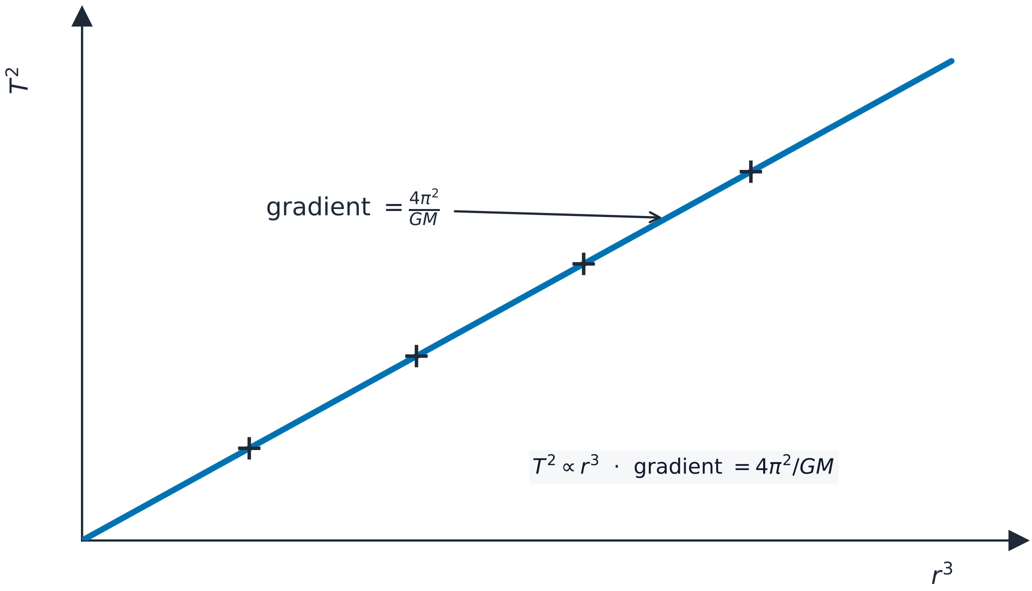
Worked example. A satellite orbits the Earth in a circular orbit of radius $r = 7.0 \times 10^6$ m. Find its orbital speed. (For the Earth, $GM = 4.0 \times 10^{14}$ m^3 s^{-2}.)

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{4.0 \times 10^{14}}{7.0 \times 10^6}} \approx 7.6 \times 10^3 \text{ m s}^{-1}.$$

The **period** 周期 follows from $T = 2\pi r/v$:

$$T = 2\pi\sqrt{\frac{r^3}{GM}}, \quad \text{so} \quad T^2 = \frac{4\pi^2}{GM} \cdot r^3.$$

This is **Kepler's third law** 开普勒第三定律 for circular orbits: $T^2 \propto r^3$. A plot of T^2 against r^3 is a straight line through the origin with gradient $4\pi^2/(GM)$, so orbital data gives the central mass.



Kepler's third law: $T^2 \propto r^3$, a straight line through the origin

Geostationary orbit

A **geostationary** 地球同步 satellite:

- stays directly above the same point on the Earth (so a fixed dish always points at it),
- has a period of **24 hours** (the same as the Earth's rotation),
- orbits **west to east** (the same way the Earth turns),
- must be directly above the **equator** 赤道.

It must have the same **angular speed** 角速度 as the Earth, in the same direction, in the equatorial plane (or it would drift north–south during the day). From $T = 24$ h and $T^2 = 4\pi^2 r^3/(GM)$, the radius is $r \approx 4.2 \times 10^7$ m (about 3.6×10^7 m above the surface).

Gravitational potential

Gravitational potential 引力势 ϕ at a point is the **work done per unit mass** in bringing a small test mass from **infinity** 无穷远 to that point:

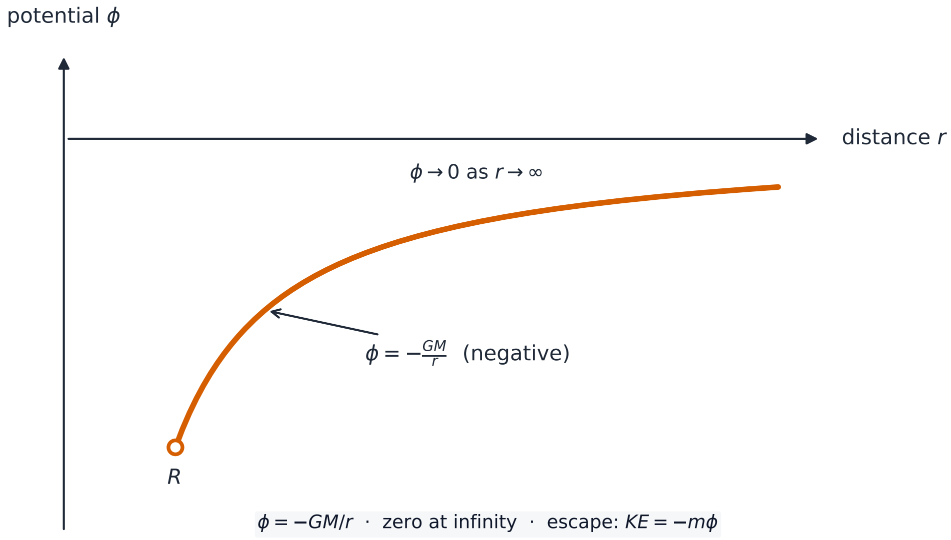
$$\phi = \frac{W}{m}.$$

Unit: J kg^{-1}.

The potential is taken as **zero at infinity**. As the test mass falls in towards the source, gravity does the work for you, so ϕ is **negative** everywhere except at infinity. For a point mass M at distance r:

$$\phi = -\frac{GM}{r}.$$

ϕ is a **scalar 标量**. For several masses, add the potentials.



The gravitational potential well: $\phi = -GM/r$ is negative, rising to zero at infinity

Gravitational potential energy of two point masses

If a test mass m sits where the potential is ϕ, the **gravitational potential energy 重力势能** of the pair is

$$E_{\mathrm{P}} = m\phi = -\frac{GMm}{r}.$$

Like the potential, E_{P} is **negative** and reaches zero only at infinite separation. Closer masses have more negative potential energy (more tightly bound).

Link with $\Delta E_{\mathbf{P}} = mg\Delta h$

For small height changes near the surface, r barely changes, so $\Delta E_{\mathrm{P}} \approx mg\Delta h$. For large changes (a satellite moving to a higher orbit) use $-GMm/r$ at each radius and take the difference:

$$\Delta E_{\mathrm{P}} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad (r_2 > r_1),$$

which is positive (energy must be supplied to raise the satellite).

Escape velocity (from conservation of energy)

To escape from radius r to infinity, an object's **kinetic energy 动能** must equal the size of its gravitational potential energy:

$$\tfrac{1}{2}mv_{\mathrm{esc}}^2 = \frac{GMm}{r}, \qquad v_{\mathrm{esc}} = \sqrt{\frac{2GM}{r}}.$$

At the Earth's surface, the **escape velocity 逃逸速度** is ≈ 11 km s^{-1}. It does not depend on the object's mass.

Worked example. Find the escape velocity from the Earth's surface. (For the Earth, $GM = 4.0 \times 10^{14}$ m^3 s^{-2}, $R = 6.4 \times 10^6$ m.)

$$v_{\mathrm{esc}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(4.0 \times 10^{14})}{6.4 \times 10^6}} \approx 1.1 \times 10^4 \text{ m s}^{-1} \ (= 11 \text{ km s}^{-1}).$$

Exam tips

- **Newton's law of gravitation** $F = GMm/r^2$ (inverse-square); field strength $g = GM/r^2$.
- Distinguish **gravitational potential** ($\phi = -GM/r$, always negative, zero at infinity) from field strength.
- For an orbit set gravity = centripetal force to get $T^2 \propto r^3$; a geostationary orbit has $T = 24$ h.