Work, energy and power

A-Level Physics

Work, energy and power

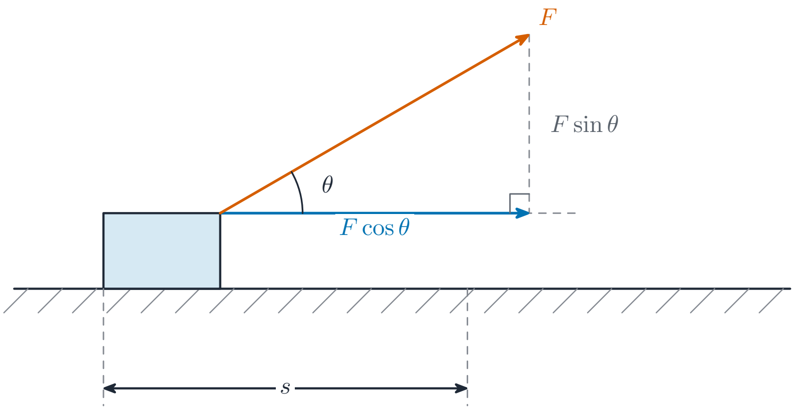
Wind turbines transfer the kinetic energy of the wind into electrical energy.

Work done by a force

Work 功 is done when a force moves its point of contact along the line of the force. For the one-mark definition write: **work done is the product of the force and the distance moved in the direction of the force.** The work done by a constant force F that causes a **displacement 位移** s is

$$W = F \cdot s \cdot \cos\theta,$$

where θ is the angle between the force and the displacement. Only the **component 分量** of the force along the displacement does work.



Only the component of the force along the displacement ($F\cos\theta$) does work

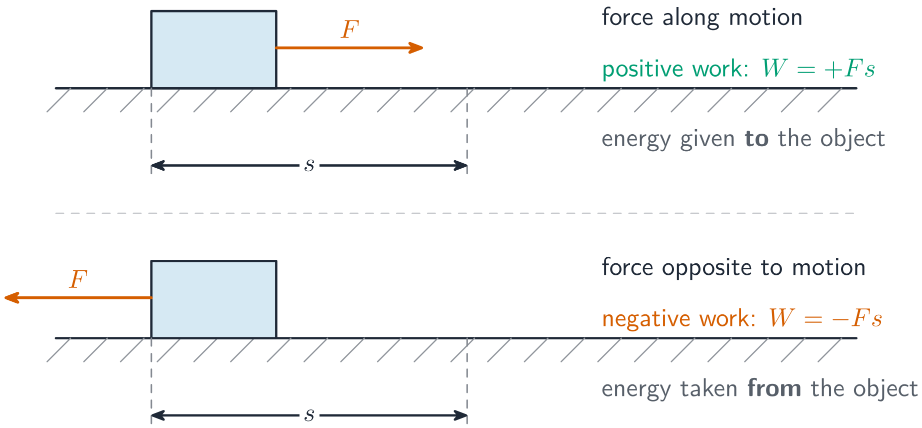
Worked example. A child pulls a sledge 5.0 m across the snow with a rope, using a force of 20 N at 60° to the ground. Find the work done by the rope.

$$W = Fs\cos\theta = 20 \times 5.0 \times \cos 60^\circ = 20 \times 5.0 \times 0.50 = 50 \text{ J}.$$

Unit: J = N m. Work is a **scalar 标量**.

Special cases:

- force in the same direction as the motion ($\theta = 0$): $W = Fs$, positive work, **energy 能量** given **to** the object.
- force at right angles to the motion ($\theta = 90^\circ$): $W = 0$. The **normal contact force 支持力** on a car on a flat road does no work.
- force opposite to the motion ($\theta = 180^\circ$): $W = -Fs$, negative work, energy taken **from** the object (for example **friction 摩擦力**).

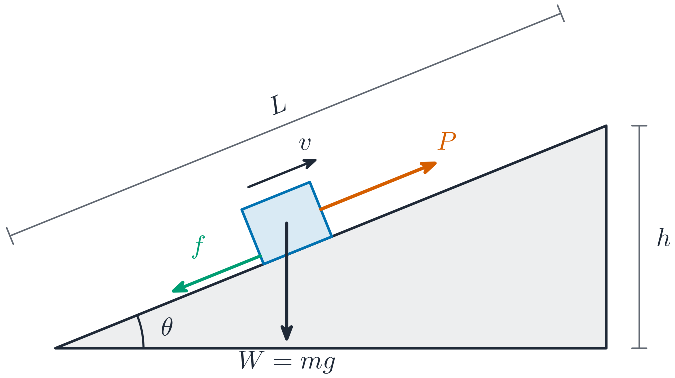


Positive work ($W = +Fs$): force along the motion (top). Negative work ($W = -Fs$): force opposite to the motion, e.g. friction (bottom)

For an object moving up a slope at angle α to the **horizontal 水平**, the work done against gravity in rising a height h is mgh, while the work done by a horizontal push over the slope length L uses $\cos\alpha$.

Two multiple-choice tests of the definition: no work is done on an object that slides at constant velocity along a frictionless surface (no force along the motion), or on one that is simply held still (no distance moved); work is done when a force lifts it. And when a box is pushed at constant velocity across a rough floor, the work done by the push is $F \times d$ for the whole distance, however the path is split into stages.

Worked example. A block of mass 4.9 kg is pushed at constant velocity up a rough slope of length 8.0 m that rises 1.5 m, by a force of 12 N acting along the slope. Find the work done against friction.



A box pushed up a rough slope at constant velocity: the work done by the push pays for the gain in GPE, which needs only the vertical rise h, and for the work against friction along the whole length L

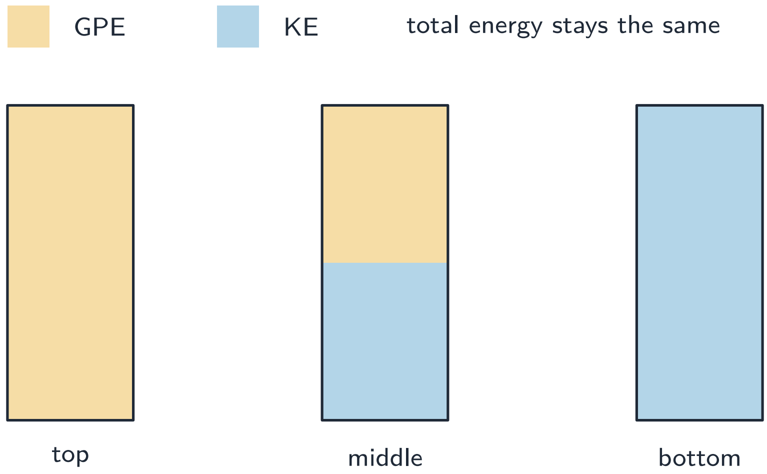
Constant velocity means no change in kinetic energy, so the work done by the push is shared between the gain in gravitational potential energy and the work done against friction. Work by the push: $12 \times 8.0 = 96$ J. Gain in GPE, using the vertical rise: $4.9 \times 9.81 \times 1.5 = 72$ J. So the work done against friction is $96 - 72 = 24$ J, and the friction force is $24/8.0 = 3.0$ N. Using the slope length in mgh is the commonest error on this question.

Conservation of energy

Energy is **never made or destroyed** — it only changes from one form to another, or moves from one object to another. In a **closed system** 封闭系统, the total energy stays constant. This is **conservation of energy** 能量守恒, stated for the marks as: **energy cannot be created or destroyed; it can only be transferred from one form to another** (so the total energy of a closed system is constant).

When you write an energy equation, list every form the energy starts as and ends as. Common forms in this syllabus: kinetic, gravitational potential, **elastic potential energy** 弹性势能, **electrical** 电能, thermal, sound, **chemical energy** 化学能.

A ball rolling down a **frictionless** 无摩擦 **ramp** 斜坡 turns **gravitational potential energy** 重力势能 into **kinetic energy** 动能: $mgh = \frac{1}{2}mv^2$, so $v = \sqrt{2gh}$. With friction, some of this energy becomes **thermal energy** 热能 of the ramp and the air.



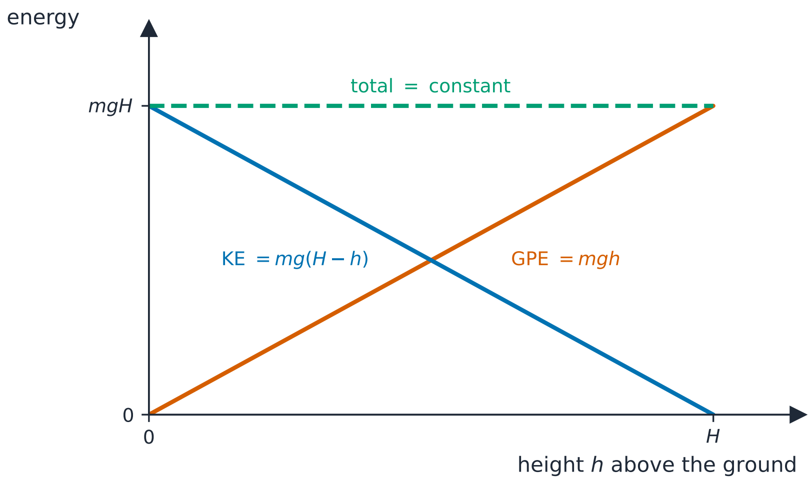
On a frictionless ramp, GPE turns into KE while the total energy stays constant

Worked example. A ball is released from rest at the top of a smooth ramp 1.2 m high. Find its speed at the bottom (take $g = 9.81$ m s^{-2}).

All the gravitational potential energy becomes kinetic energy, so $v = \sqrt{2gh}$ (the mass cancels):

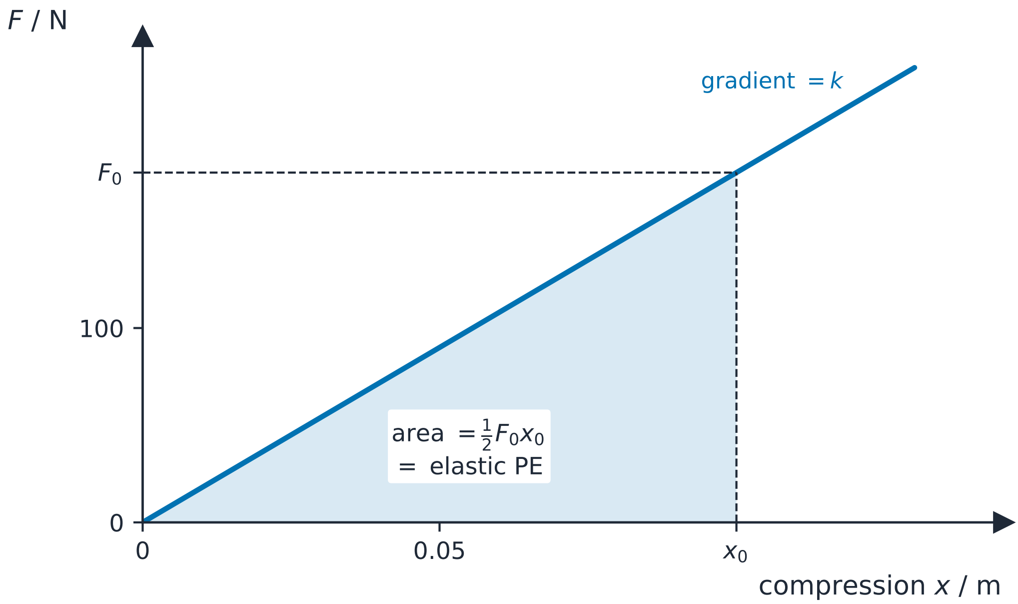
$$v = \sqrt{2 \times 9.81 \times 1.2} \approx 4.9 \text{ m s}^{-1}.$$

The same idea reads a graph. For a ball falling from height H with air resistance negligible, the GPE falls in a straight line with height (mgh), the KE rises in a straight line ($mg(H - h)$), and the two add to the constant mgH at every height.



A falling ball: GPE and KE both change in a straight line with height, and their sum is constant

The energy chain can pass through a spring. A block that hits a spring with kinetic energy 110 J while sliding a little downhill, so that its GPE falls by a further 20 J before the spring stops it, stores $110 + 20 = 130$ J of elastic potential energy at the greatest compression, if nothing is lost to friction. The spring's force–compression graph then gives the compression: the energy stored is the **area** under the line, $\frac{1}{2}F_0x_0$ for a spring that obeys Hooke's law (topic 6), so a maximum force of 2600 N means $x_0 = 2 \times 130/2600 = 0.10$ m. At that instant the resultant force on the block is the spring force minus the component of the weight down the slope, and its acceleration follows from $F = ma$ (topic 3).



The energy stored in a spring is the area under its force–compression graph: $\frac{1}{2}F_0x_0$, which equals $\frac{1}{2}kx_0^2$

Efficiency

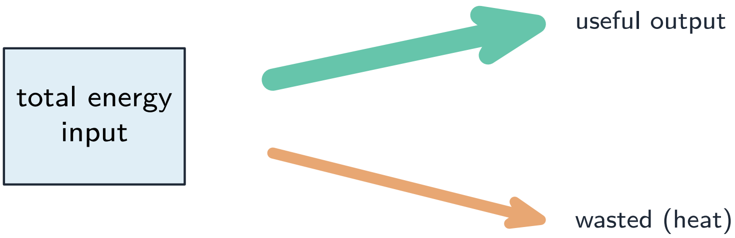
The **efficiency** 效率 of a system is

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%.$$

The same idea with power:

$$\text{efficiency} = \frac{\text{useful power output}}{\text{total power input}} \times 100\%.$$

Efficiency is always less than 100% in a real system, because some input energy becomes ”useless” forms — usually thermal energy. If a total energy E is supplied and an amount Q is wasted, the useful output is $E - Q$ and the efficiency is $(E - Q)/E$; a motor rated at 1.7 kW input with efficiency 53% delivers $0.53 \times 1.7 = 0.90$ kW of useful output.



$$\text{efficiency} = \frac{\text{useful output}}{\text{total input}} \times 100\%$$

Efficiency: only part of the energy input leaves as useful output; the rest is wasted, usually as heat

For an electric motor lifting a load with efficiency η at **voltage** 电压 V and **current** 电流 I, the useful output power is ηVI. From this you can find a force, a lifting speed, or a **tension** 张力.

Worked example. An electric motor lifts a 50 kg load at a steady 0.40 m s^{-1} while drawing 250 W of electrical power. Find its efficiency (take $g = 9.81$ m s^{-2}).

The useful output power is $P = mgv = 50 \times 9.81 \times 0.40 = 196$ W, so

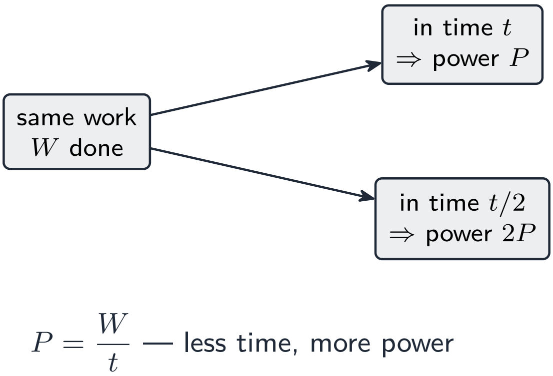
$$\text{efficiency} = \frac{196}{250} \times 100\% \approx 78\%.$$

Worked example. A motor raises a block with a force of 240 N through a vertical distance of 12 m in 60 s. The input power to the motor is 900 W. Find the useful output power and the efficiency.

Work done on the block: $W = Fd = 240 \times 12 = 2880$ J. Useful output power: $P = W/t = 2880/60 = 48$ W. Efficiency: $48/900 \times 100\% = 5.3\%$. Most of the input goes into thermal energy in the motor and its cable. Turned round, a lift motor of useful output 20 kW raising 1500 kg through 20 m takes $t = mgh/P = 1500 \times 9.81 \times 20/20\,000 = 15$ s.

Power

Power 功率 is the rate of doing work, or the rate of transferring energy:



The same work done in less time means more power

$$P = \frac{W}{t} = \frac{\Delta E}{\Delta t}.$$

Unit: $\mathrm{W} = \mathrm{J\ s^{-1}}$. Power is a scalar.

Power, force and velocity

For an object moving at **velocity** 速度 v with a force F along the direction of motion, in a short time Δt the displacement is $v\,\Delta t$ and the work done is $Fv\,\Delta t$. Dividing by Δt:

$$P = Fv.$$

This is one of the most useful results in mechanics. A "derive $P = Fv$" answer needs exactly those three lines: $P = W/t$, $W = Fs$ for a force along the motion, and $s/t = v$; then use it to solve problems in which a vehicle or a load moves at constant speed.

Worked example. A car travels at a steady 25 m s^{-1} against a total resistive force of 600 N. Find the output power of its engine.

At constant speed the driving force equals the resistive force, so

$$P = Fv = 600 \times 25 = 15\,000\ \mathrm{W} = 15\ \mathrm{kW}.$$

- For a car at constant velocity v on a flat road, the engine power must balance the total resistive force: $P = F_{\text{resist}} \cdot v$. If the **drag** 阻力 grows with v^2, doubling the speed roughly **quadruples** the power needed.
- For lifting a **weight** 重力 mg straight up at constant speed v, the useful output power is $P = mg \cdot v$.
- For an aircraft hovering at a fixed height, the lift force equals the weight, and a large power is needed because air must be pushed downwards all the time.
- A crane raising 600 kg at a steady 12 m per minute lifts at $v = 0.20$ m s^{-1}, so its useful output power is $mgv = 600 \times 9.81 \times 0.20 = 1.2$ kW. Convert the speed to m s^{-1} first.
- A sailboat pushed by a constant wind force at constant velocity must also feel an equal resistive force from the water: constant velocity means zero resultant force, so the wind's power Fv is all going into work against the water.

Gravitational potential energy

In a **uniform** gravitational field (close to a planet's surface), the change in gravitational potential energy of **mass** 质量 m rising or falling through a height Δh is

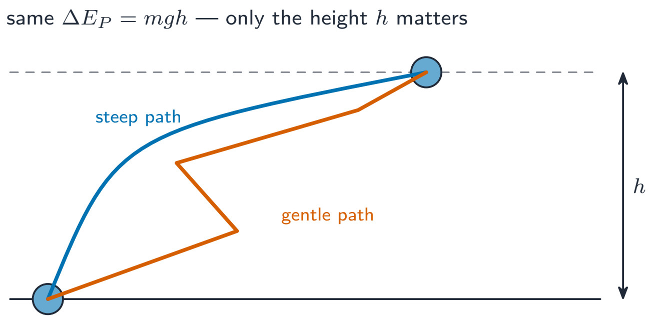
$$\Delta E_{\text{P}} = mg\Delta h.$$

Where it comes from

The work done **against** gravity to raise a mass m slowly (no change in kinetic energy) through height Δh equals the gravitational potential energy gained:

- the gravitational force on the mass is mg downwards,
- the force needed to lift it slowly is mg upwards,
- the work done by this force is $W = F \cdot s = mg \cdot \Delta h$,
- this work becomes ΔE_{P}.

So $\Delta E_{\text{P}} = mg\Delta h$. To use it you need m and Δh (and g). You do not need speed or time. The two-mark derivation wants the same four lines, with the condition stated: the block is raised **at constant speed**, so the lifting force equals the weight mg, and the work done by that force over the vertical distance Δh is $mg\Delta h$, which is the gain in gravitational potential energy. A 100 g object falling 10 m loses $0.100 \times 9.81 \times 10 = 9.8$ J, about 10 J; the mass must be in kilograms.



Gravitational PE depends only on the height risen, not the path taken

Kinetic energy

The kinetic energy of an object of mass m moving at speed v is

$$E_{\mathrm{K}} = \tfrac{1}{2}mv^2.$$

Where it comes from

Apply a resultant force F to a mass m that starts at rest. It speeds up evenly from 0 to v over a displacement s. From $v^2 = u^2 + 2as$ with $u = 0$,

$$s = \frac{v^2}{2a}.$$

The work done on the mass is

$$W = F \cdot s = ma \cdot \frac{v^2}{2a} = \tfrac{1}{2}mv^2.$$

All this work becomes kinetic energy, so $E_{\mathrm{K}} = \frac{1}{2}mv^2$. In the exam, state the assumptions as you go: the object starts from rest, the force is constant, so the acceleration is uniform, and $F = ma$ is Newton's second law.

Because $E_{\mathrm{K}} \propto v^2$, quadrupling the speed multiplies the kinetic energy by sixteen: an object with 1500 J at 10 m s^{-1} has 24 000 J at 40 m s^{-1}. The same square is why a stopping force is found from energy: a ball of mass 1.2 kg moving at 3.0 m s^{-1} that is stopped by a cushion over 0.020 m loses $\frac{1}{2} \times 1.2 \times 3.0^2 = 5.4$ J, so the average force on it is $5.4/0.020 = 270$ N: the work done by the cushion equals the kinetic energy lost.

Kinetic energy and momentum

Combining $p = mv$ and $E_\mathrm{K} = \frac{1}{2}mv^2$:

$$E_\mathrm{K} = \frac{p^2}{2m}.$$

This is handy when the **momentum** 动量 is given but not the velocity. For a momentum change from p_1 to p_2 at constant mass, the change in kinetic energy is $(p_2^2 - p_1^2)/(2m)$.

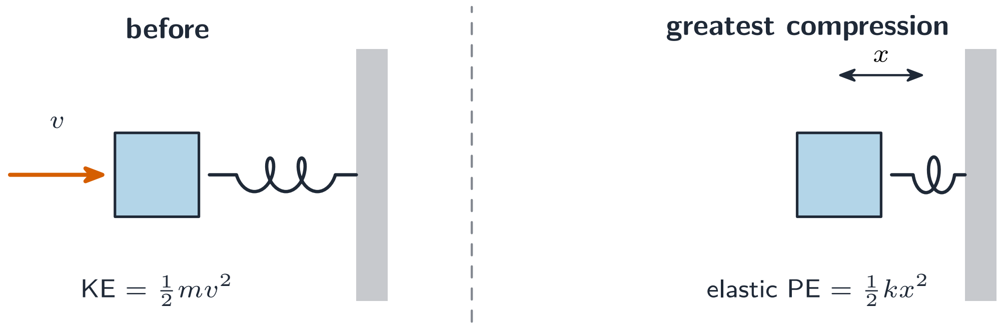
Using energy methods

A useful plan for problems that mix forces and energy:

1. **Find the start and end states.** Write the kinetic and potential energies in each.
2. **List any work done by outside forces** (friction, a push). Friction usually takes energy out; a push can add it.
3. **Conservation of energy**: $E_\mathrm{start} + W_\mathrm{in} = E_\mathrm{end} + W_\text{lost as heat etc.}$.

Examples:

- A box pushed at **constant velocity** up a ramp of length L rising by h: E_K does not change, so the work done by the push goes into ΔE_P plus the work done against friction.
- A block sliding into a **spring** 弹簧 with kinetic energy E_K on a frictionless surface: at greatest **compression** 压缩 x, all the kinetic energy has become elastic potential energy $\frac{1}{2}kx^2$ (where k is the **spring constant** 劲度系数).
- A ball dropped from height h_1 that bounces to height h_2: the ratio h_2/h_1 is the fraction of mechanical energy kept, $h_2/h_1 = (v_\mathrm{up}/v_\mathrm{down})^2$.
- A **projectile** 抛体 thrown to the same height at different angles: the final speed is the same (only the height matters); use components to get its direction.
- A **bungee jumper** 蹦极者: from the platform to the point where the cord goes taut, GPE becomes KE; from there the cord stretches and takes energy as elastic potential energy, so the KE reaches its maximum where the cord's pull first equals the weight, then falls to zero at the lowest point, where GPE lost = elastic PE stored (plus any thermal energy).



A block's kinetic energy becomes elastic potential energy as it compresses the spring

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
work done by a force	the product of the force and the distance moved in the direction of the force
principle of conservation of energy	energy cannot be created or destroyed, only transferred from one form to another, so the total energy of a closed system is constant
efficiency	the ratio of useful energy (or power) output from a system to the total energy (or power) input, usually as a percentage
power	the work done per unit time, or the rate at which energy is transferred
gravitational potential energy	the energy an object has because of its position in a gravitational field
kinetic energy	the energy an object has because of its motion
elastic potential energy	the energy stored in an object that has been stretched or compressed

Exam tips

- **Work = force × distance moved in the direction of the force** — use $Fs\cos\theta$ when they are at an angle.
- Use **conservation of energy**: loss in GPE = gain in KE (+ work done against resistance).
- **Power = work / time** $= Fv$; efficiency = useful output ÷ total input.

- GPE uses the **vertical** height gained, not the distance along a slope.

Common mistakes

- Giving the change in height as the answer to a change-in-energy question. Finish the calculation: $\Delta E_{\mathrm{P}} = mg\Delta h$.
- Using the distance along a slope as the height in $mg\Delta h$, or using mass where weight is needed. Use the vertical height gained, and check whether the quantity wanted is m in kg or W in N.
- Putting the resistive force into $P = Fv$ for a car that is accelerating. F is the driving force; only at constant speed does it equal the resistive force.
- Efficiency above 100%, or efficiency as output divided by wasted. It is useful output divided by total input, and the total input is always the larger number.
- Reading a force–extension graph's gradient when the question wants the energy. Energy stored is the area under the line, not its slope.