Physical quantities and units

A-Level Physics

Physical quantities

A **physical quantity** 物理量 has two parts: a number (its **magnitude** 大小) and a **unit** 单位. The number on its own tells you nothing. You must also say what is measured and in which unit.

Example: “the length is 1.5” is not complete. “The length is 1.5 m” is a physical quantity.

Making estimates

You should be able to **estimate** 估算 the size of the physical quantities in this syllabus. Paper 1 usually opens with a question like this. Learn these rough values:

- **mass** 质量 of an adult human: ~ 70 kg; of a car: ~ 1000 kg; of an apple: ~ 0.1 kg
- **weight** 重力 of an adult human: ~ 700 N; of an apple: ~ 1 N
- height of an adult human: ~ 1.7 m; height of a room: ~ 3 m
- walking speed: ~ 1.5 m s^{-1}; a car on a fast road: ~ 30 m s^{-1}
- speed of sound in air: ~ 340 m s^{-1}; speed of light in a **vacuum** 真空: 3.0×10^8 m s^{-1}
- **acceleration** 加速度 of **free fall** 自由落体: $g \approx 9.81$ m s^{-2}
- **density** 密度 of water: 1000 kg m^{-3}; of air: ~ 1.2 kg m^{-3}; of steel: ~ 8000 kg m^{-3}
- **atmospheric pressure** 大气压强: $\sim 1.0 \times 10^5$ Pa
- room temperature: ~ 20 °C ≈ 293 K
- power of a kettle: ~ 2 kW; of a person climbing stairs: ~ 300 W
- wavelength of visible light: $\sim 5 \times 10^{-7}$ m; diameter of an atom: $\sim 10^{-10}$ m; of a nucleus: $\sim 10^{-15}$ m

A good estimate has the right **order of magnitude** 数量级 (the right power of ten). For a human, 70 kg is a good guess; 7 kg is not.

To estimate a quantity that is not in the list, build it from ones that are. The kinetic energy of a moving car is $\frac{1}{2}mv^2 \approx \frac{1}{2} \times 1000 \times 30^2 \approx 5 \times 10^5$ J. The pressure under a standing person is 700 N/0.02 m$^2 \approx 4 \times 10^4$ Pa. Always check that the power of ten looks sensible before you move on.

SI units 国际单位制

Base units

The SI system has seven **base quantities** 基本量; five of them are used in this topic. You must know these five and their units:

- **mass** — kilogram, kg
- **length** 长度 — metre, m
- time — second, s
- **current** 电流 — ampere 安培, A
- **temperature** 温度 — kelvin 开尔文, K

Every other unit in this syllabus is built from these five.

Derived units

A **derived unit** 导出单位 is made by multiplying or dividing base units. You should be able to write any quantity in this syllabus in base units.

Build a derived unit from the equation that defines it:

- **speed** 速率 = distance / time, so its unit is m s^{-1}
- **acceleration** = change in **velocity** 速度 / time, so its unit is m s^{-2}
- **force** 力 = mass × acceleration, so its unit is kg m s^{-2}. The **newton** 牛顿 is $1\ \mathrm{N} = 1\ \mathrm{kg\ m\ s^{-2}}$.
- **work** 功 and **energy** 能量 = force × distance, so the unit is kg m^2 s^{-2}. The **joule** 焦耳 is $1\ \mathrm{J} = 1\ \mathrm{kg\ m^2\ s^{-2}}$.
- **power** 功率 = energy / time, so the unit is kg m^2 s^{-3}. The **watt** 瓦特 is $1\ \mathrm{W} = 1\ \mathrm{kg\ m^2\ s^{-3}}$.
- **pressure** 压强 and **stress** 应力 = force / area, so the unit is kg m^{-1} s^{-2}. The **pascal** 帕斯卡 is $1\ \mathrm{Pa} = 1\ \mathrm{kg\ m^{-1}\ s^{-2}}$.

When a question asks for the SI base units of a quantity, replace each named unit with its base units, then simplify. Example: the SI base units of the watt are kg m^2 s^{-3}.

Checking that the units match

An equation is **homogeneous** 量纲一致 when both sides have the same base units. In plain words: the units on both sides match.

Write each side in base units and compare. Take the equation $v^2 = u^2 + 2as$:

- left side: $(\mathrm{m\ s^{-1}})^2 = \mathrm{m^2\ s^{-2}}$

- right side, first term: $(\mathrm{m\ s^{-1}})^2 = \mathrm{m^2\ s^{-2}}$
- right side, second term: $\mathrm{m\ s^{-2}} \cdot \mathrm{m} = \mathrm{m^2\ s^{-2}}$

Both sides give $\mathrm{m^2\ s^{-2}}$, so the units match.

Be careful: matching units do **not** prove the whole equation is correct. It could still have a wrong number, or a missing factor of 2. But if the units do **not** match, the equation is wrong for sure.

Worked example. The drag force on a falling ball is given by $F = kv^2$, where v is the speed. Find the SI base units of the constant k.

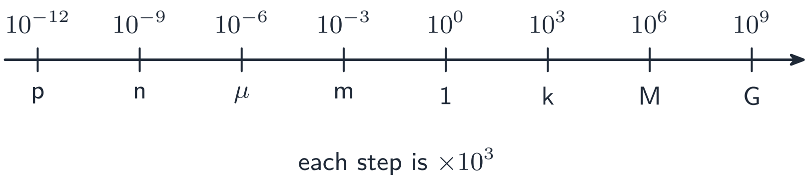
Rearrange: $k = F/v^2$. In base units, F is $\mathrm{kg\ m\ s^{-2}}$ and v^2 is $\mathrm{m^2\ s^{-2}}$, so

$$k : \frac{\mathrm{kg\ m\ s^{-2}}}{\mathrm{m^2\ s^{-2}}} = \mathrm{kg\ m^{-1}}.$$

A multiple-choice question often asks "which equation could be correct?". Check the base units of each option; only a homogeneous equation can be correct. A pure number (like 2, π or $\frac{1}{2}$) has no unit, so it never changes the check.

Prefixes

A **prefix** 词头 is a letter put in front of a unit to make it bigger or smaller by powers of ten. You must know these:



The SI prefixes climb in steps of a thousand, from pico to giga

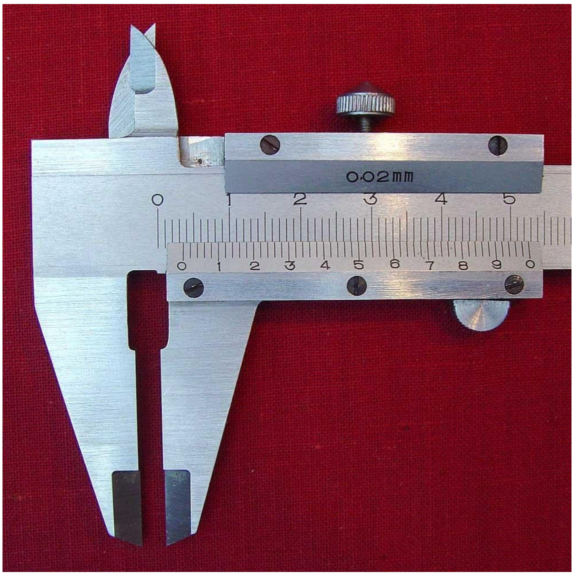
Prefix	Symbol	Factor
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

To change a prefixed unit into base units, replace the prefix with its factor, then simplify. Example: change 0.25 kN mm^{-2} into N m^{-2}:

$$0.25\ \text{kN mm}^{-2} = 0.25 \times \frac{10^3\ \text{N}}{(10^{-3}\ \text{m})^2} = 0.25 \times \frac{10^3}{10^{-6}}\ \text{N m}^{-2} = 2.5 \times 10^8\ \text{N m}^{-2}.$$

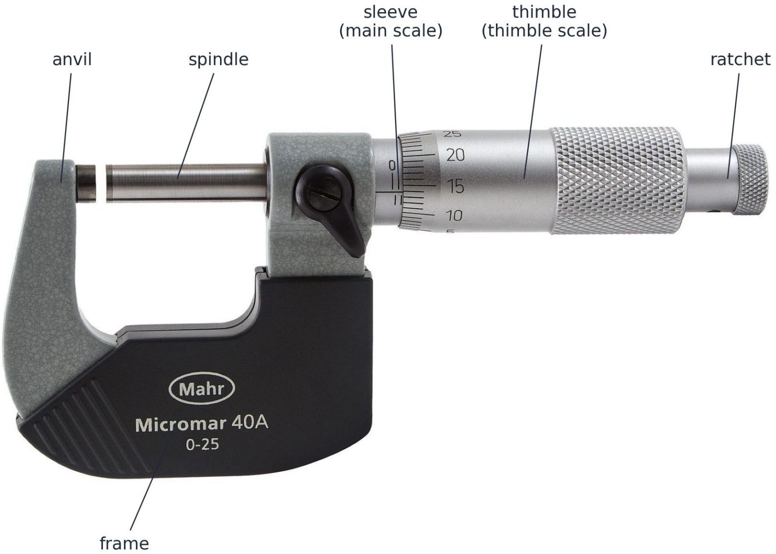
Take special care with squared units like mm^2: you must square the factor too.

Errors and uncertainties

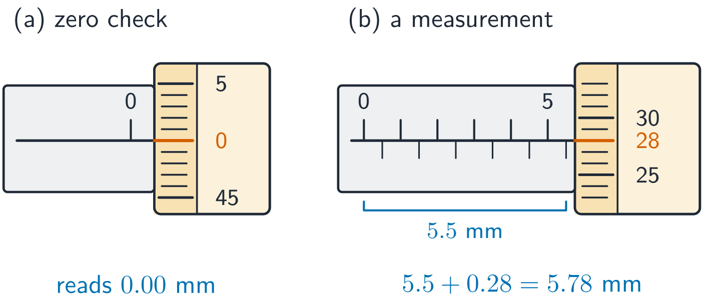


A vernier caliper measures length precisely, with a small uncertainty.

Every **measurement** 测量 has some **uncertainty** 不确定度 — we are never fully sure of the value. A good experimenter knows where the uncertainty comes from, makes a fair estimate of it, and carries it through to the final answer.



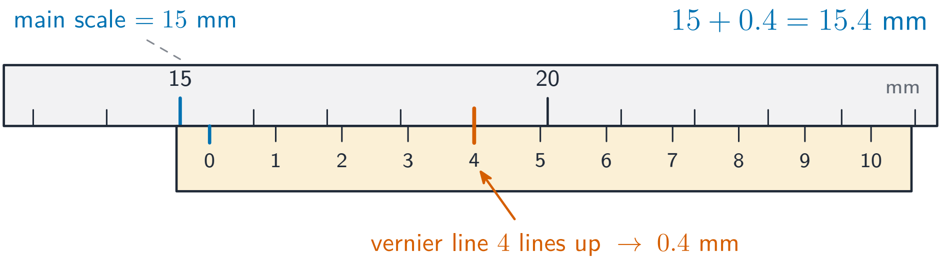
The main parts of a real micrometer screw gauge, which measures to the nearest 0.01 mm



Reading a micrometer: add the main scale reading to the thimble reading

Reading a real micrometer: read the mm and half-mm on the sleeve, then add the thimble scale

Vernier calipers measure to the nearest 0.1 mm — the sliding scale gives the extra digit

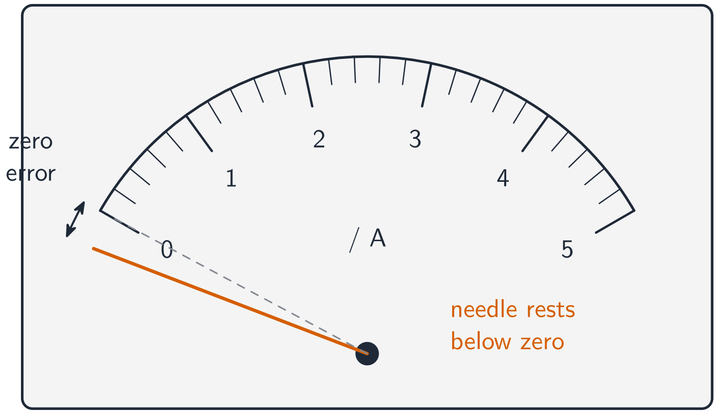


Reading a vernier caliper: whole millimetres from the main scale, plus the tenths from the vernier line that lines up

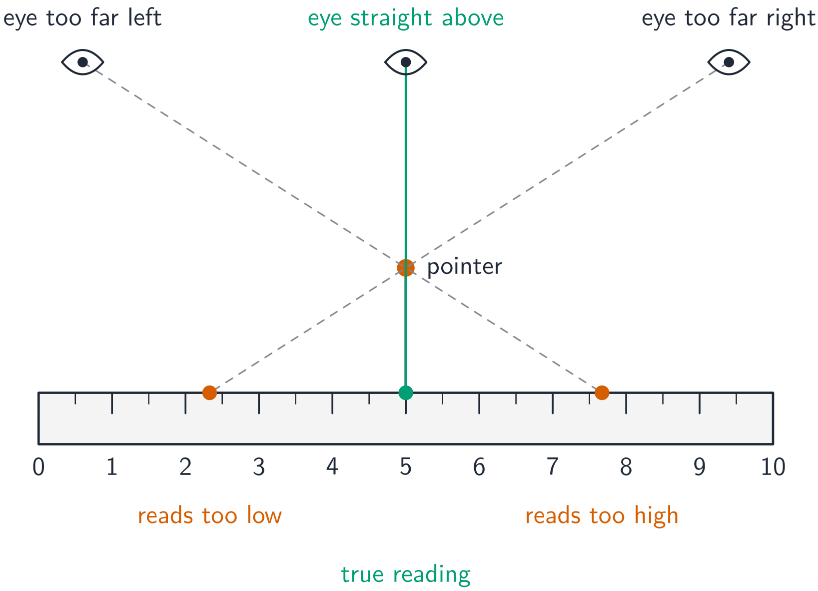
Systematic and random errors

A **systematic error** 系统误差 changes every reading by the same amount, in the same direction. You cannot find it by repeating the measurement. Common causes:

- a **zero error** 零点误差 (the scale does not read zero when the true value is zero)
- a **calibration** 校准 error (the scale itself is wrong)
- **parallax** 视差 (your eye is always to one side of the scale)



An ammeter with a zero error: the needle reads below zero before any current flows



Parallax error: different viewing angles give different scale readings

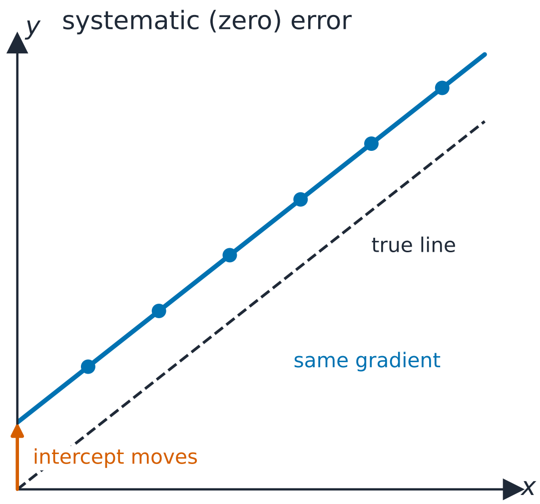
A systematic error makes the **accuracy** 准确度 worse, but it does not change the

precision 精密度.

A **random error** 随机误差 makes readings jump above and below the true value, with no pattern. Causes include how carefully you read the scale, changing conditions, and the smallest step the **instrument** 仪器 can show. If you repeat the measurement many times and take the **mean** 平均值 (the average), random errors partly cancel out.

A random error makes the precision worse. But with enough repeats, the mean can still be accurate.

On a graph the two errors look different. A systematic error moves every point by the same amount, so the line of best fit keeps its gradient but no longer passes through the origin: the **intercept** 截距 changes. Random errors scatter the points above and below the line; a best-fit line drawn through the middle of the scatter still gives a reliable gradient.



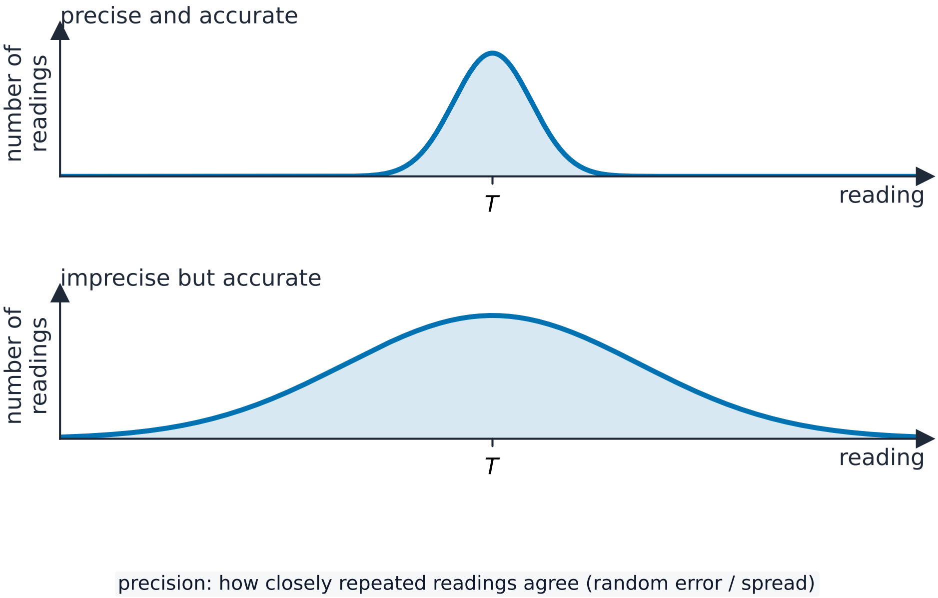
A systematic error shifts the whole line, so the intercept moves; random errors scatter the points about the line

Error	How to reduce it
systematic	check for a zero error and subtract it; calibrate the instrument against a known standard; read the scale from directly in front
random	repeat the reading and take the mean; use an instrument with smaller divisions; time many oscillations instead of one

Precision and accuracy

Precision is how close repeated readings are to each other. Precise readings are grouped very close together.

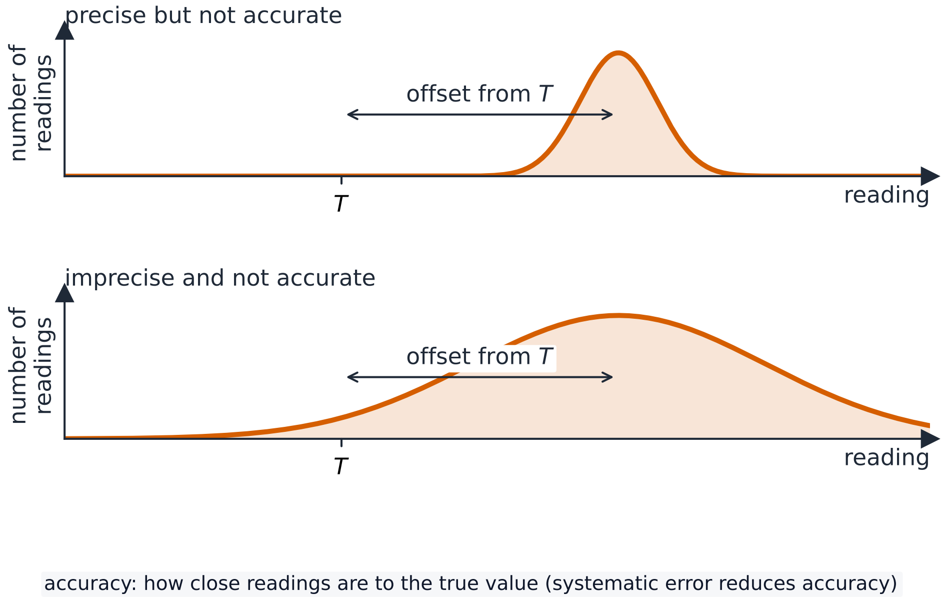
Accuracy is how close a reading (or the mean of several readings) is to the true value.



Precision: how narrow the distribution is around the true value T

A set of readings can be:

- precise and accurate — close together and near the true value
- precise but not accurate — close together, but away from the true value (a systematic error)
- accurate but not precise — spread out, but the mean is near the true value
- neither — spread out and away from the true value



Accuracy: whether the peak of the distribution is centred on the true value T

When a question gives a table of repeated readings, look at the **spread** (precision) and the **mean** (accuracy) separately.

Estimating the uncertainty in a reading

Before you combine uncertainties, you need a sensible uncertainty for each raw reading:

- A single reading on an analogue scale: half the smallest **division** 分度 (a metre rule reads to 1 mm, so one reading is ± 0.5 mm). A length needs two readings, one at each end, so its uncertainty is ± 1 mm.
- A digital meter: ± 1 in the last digit shown (a stopwatch showing 2.47 s is ± 0.01 s).
- A hand-timed measurement: your **reaction time** 反应时间 matters more than the display. Allow about ± 0.1 to 0.2 s, and time many oscillations instead of one, so the same uncertainty is shared between all of them.
- Repeated readings: the uncertainty is **half the range** 极差的一半. Three timings of 2.42, 2.48 and 2.45 s have a mean of 2.45 s and a range of 0.06 s, so $t = (2.45 \pm 0.03)$ s.

Be honest. If the edge of a shadow is hard to see, the uncertainty in its position is several millimetres, however fine the scale. Examiners do not accept "the smallest division" as the uncertainty of a difficult reading.

Uncertainty in a derived quantity

A measurement is often written as $x \pm \Delta x$. Here Δx is the **absolute uncertainty 绝对不确定度**. The **percentage uncertainty 百分比不确定度** is

$$\text{percentage uncertainty in } x = \frac{\Delta x}{|x|} \times 100\%.$$

A **derived quantity 导出量** is one you calculate from measured values. Its uncertainty is found by simple rules:

- **Adding or subtracting** — add the absolute uncertainties. If $y = a+b$ or $y = a-b$, then $\Delta y = \Delta a + \Delta b$.
- **Multiplying or dividing** — add the percentage uncertainties. If $y = \frac{a \cdot b}{c}$, then

$$\frac{\Delta y}{|y|} = \frac{\Delta a}{|a|} + \frac{\Delta b}{|b|} + \frac{\Delta c}{|c|}.$$

- **Powers** — multiply the percentage uncertainty by the power. If $y = a^n$, then $\frac{\Delta y}{|y|} = |n| \cdot \frac{\Delta a}{|a|}$.

Worked example. A ball's **diameter 直径** is measured as $d = (5.26 \pm 0.02)$ cm. The **volume 体积** of a sphere is $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(d/2)^3$, so $V \propto d^3$ (V depends on d cubed).

The percentage uncertainty in d is

$$\frac{0.02}{5.26} \times 100\% \approx 0.38\%.$$

Because $V \propto d^3$, the percentage uncertainty in V is three times this, about 1.14%. The volume is $\frac{4}{3}\pi(2.63)^3 \approx 76.2\ \text{cm}^3$. So the absolute uncertainty is $0.0114 \times 76.2 \approx 0.87\ \text{cm}^3$. The final answer is $V = (76.2 \pm 0.9)\ \text{cm}^3$.

Do two values agree? Practical questions often ask whether two calculated values of a constant support a suggested relationship. Do not just say "they are close". Work out the **percentage difference 百分比差异** between them,

$$\text{percentage difference} = \frac{|k_1 - k_2|}{\text{mean of } k_1 \text{ and } k_2} \times 100\%,$$

and compare it with the percentage uncertainty in k (or with the criterion the question gives, often 10%). If the difference is smaller, the two values agree within the uncertainty and the relationship is supported. If it is larger, they do not.

Significant figures

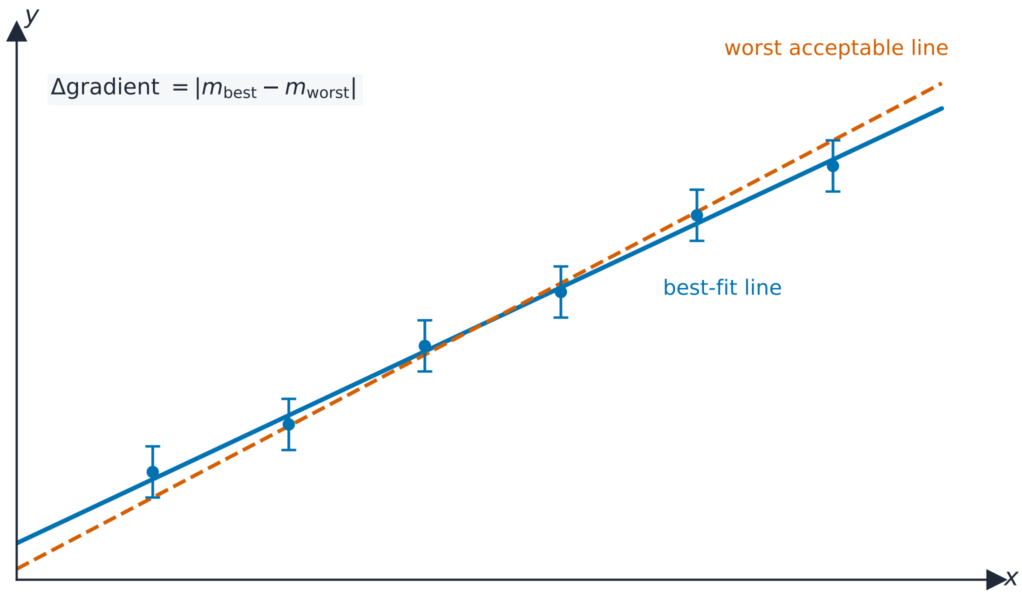
When you write a calculated quantity, give it the **same number of significant figures** 有效数字 as the least precise measurement you used — usually two or three in this syllabus. Too many significant figures makes the answer look more exact than it really is. Too few loses useful information.

Practical papers ask you to **justify** the number you chose. Name the raw readings: "a is given to 2 significant figures because L and t were each measured to 2 significant figures." The phrase "because of the raw data" on its own does not earn the mark. In a table of results, judge each row from the least precise raw reading in that row; do not force every row to the same number of figures.

Uncertainties on a graph

Paper 5 asks you to carry uncertainties through a graph:

- Plot each point with an **error bar** 误差棒 whose length shows the absolute uncertainty in that value.
- Draw the **line of best fit** 最佳拟合直线 through the points, then the **worst acceptable line** 最差可接受直线: the steepest (or shallowest) straight line that still passes through every error bar.
- The uncertainty in the gradient is the difference between the two gradients, $\Delta m = |m_{\text{best}} - m_{\text{worst}}|$. The uncertainty in the intercept is found the same way.
- When you plot a logarithm, the absolute uncertainty in $\ln x$ is $\Delta x/x$ (and in $\lg x$ it is $0.434\,\Delta x/x$). Work it out for the largest and smallest values of x separately.



The worst acceptable line still passes through every error bar; the gradient uncertainty is the difference between the two gradients

A percentage uncertainty in the gradient then flows into any quantity you calculate from it, using the rules above.

Scalars and vectors

A **scalar** 标量 has size only. A **vector** 矢量 has both size and direction.

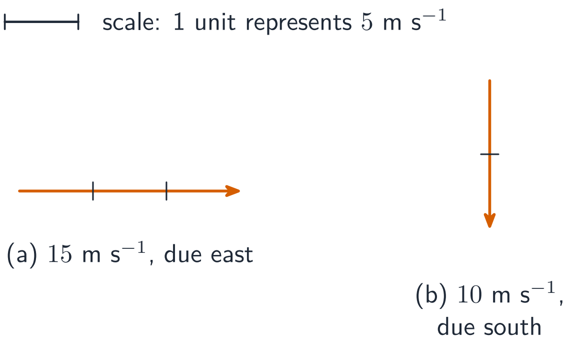
Examples from the syllabus:

- scalars: mass, time, temperature, energy, work, power, distance, speed, pressure, density, **electric charge** 电荷
- vectors: **displacement** 位移, velocity, acceleration, force (including weight), **momentum** 动量

Quick test: if it makes sense to ask “in which direction?”, the quantity is a vector. You cannot ask “in which direction is the temperature?”, so temperature is a scalar. You can ask “in which direction is the velocity?”, so velocity is a vector.

Adding and subtracting vectors

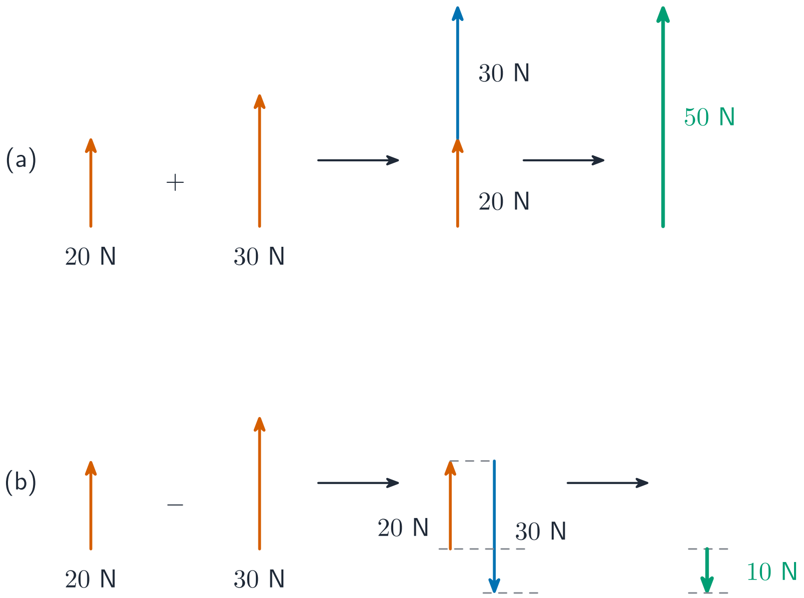
A vector is drawn as an arrow: the **direction** of the arrow gives the direction of the quantity, and the **length** of the arrow (drawn to scale) gives the magnitude.



Vectors represented as arrows drawn to scale

To add two **coplanar** 共面 vectors (vectors in the same flat plane), draw them **tip to tail**. The **resultant** 合矢量 goes from the tail of the first arrow to the tip of the second.

To find $\vec{X} - \vec{Y}$, add the reverse of $\vec{Y}$: $\vec{X} + (-\vec{Y})$. The reverse of $\vec{Y}$ has the same size as $\vec{Y}$ but points the opposite way.



Adding and subtracting parallel vectors

If the two vectors are at right angles (90°), the size of the resultant is

$$|\vec{R}| = \sqrt{X^2 + Y^2},$$

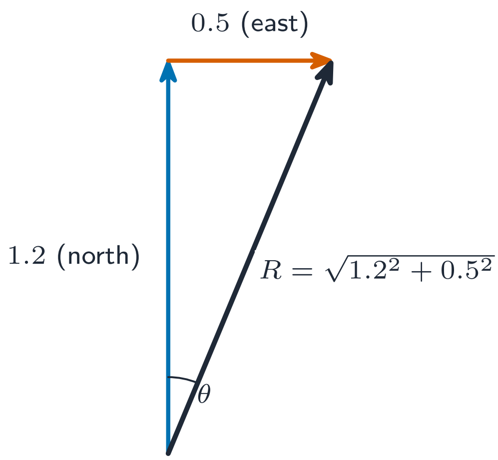
and its direction comes from $\tan\theta = Y/X$.

Worked example. A swimmer heads north at 1.2 m s^{-1} across a river that flows east at 0.5 m s^{-1}. Find the size and direction of the resultant velocity.

The two velocities are perpendicular, so

$$|\vec{R}| = \sqrt{1.2^2 + 0.5^2} = \sqrt{1.69} = 1.3 \text{ m s}^{-1},$$

at an angle $\tan\theta = 0.5/1.2$, giving $\theta \approx 23°$ east of north.



Two perpendicular vectors add to a resultant of size $\sqrt{X^2 + Y^2}$ at angle θ

If two vectors have the same size F with an angle 2α between them, the resultant has size $2F\cos\alpha$ and lies along the line that cuts the angle in half.

Worked example. Two forces of 6.0 N act on an object with 60° between them. Find the resultant.

Here $2\alpha = 60°$, so $\alpha = 30°$ and $R = 2 \times 6.0 \times \cos 30° = 10.4$ N, along the line halfway between the two forces. Resolving gives the same answer: each force has a component $6.0\cos 30°$ along that line, and their components at right angles to it cancel.

For two vectors at any other angle, either make a **scale drawing** **按比例作图** of the vector triangle and measure the resultant, or resolve both vectors into perpendicular components, add the components, and recombine with $\sqrt{X^2 + Y^2}$.

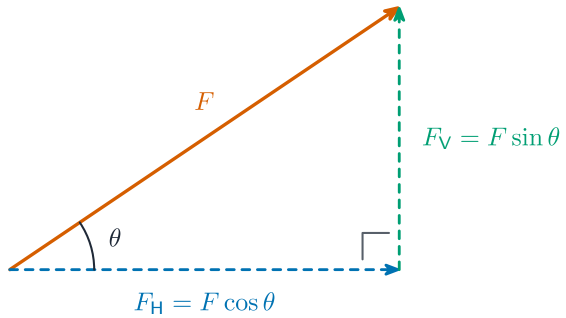
Worked example. A ball moving east at 5.0 m s^{-1} is hit so that it moves north at 5.0 m s^{-1}. Find its change in velocity.

A change in velocity is a vector subtraction: $\Delta\vec{v} = \vec{v}_{\text{final}} - \vec{v}_{\text{initial}}$. Draw 5.0 m s^{-1} north, then add the reverse of the initial velocity, 5.0 m s^{-1} west. The two are perpendicular, so $|\Delta\vec{v}| = \sqrt{5.0^2 + 5.0^2} = 7.1$ m s^{-1}, pointing north-west. The speed did not change, but the velocity did. That is why a force must have acted on the ball (topic 3).

Splitting a vector into perpendicular parts

Any vector can be split into two **perpendicular** 垂直 (at right angles) **components** 分量. Usually these are **horizontal** 水平 and **vertical** 竖直, or along and across a surface. For a vector $\vec{v}$ at angle θ to the horizontal:

$$v_{\mathrm{H}} = v\cos\theta, \qquad v_{\mathrm{V}} = v\sin\theta.$$

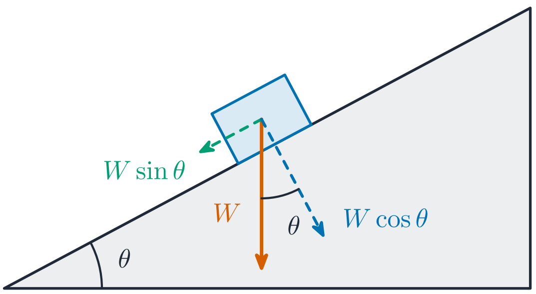


Resolving a vector into horizontal and vertical components

Choose the directions that make the problem easiest. On a slope (an **inclined plane** 斜面), split the weight into one part along the slope and one part at right angles to it:

$$W_{\parallel} = W\sin\theta, \qquad W_{\perp} = W\cos\theta,$$

where θ is the angle of the slope to the horizontal.



On a slope the weight splits into $W\sin\theta$ down the slope and $W\cos\theta$ into the slope; the angle between W and the perpendicular is the slope angle θ

You split a vector into components whenever you need to know how much of it acts in one direction. For example: the part of a force that acts along a slope, or the horizontal and vertical parts of a ball's velocity after it is thrown.

Definitions the examiner accepts

A definition question is marked against fixed wording. Learn these exactly, and give one answer only.

Term	Definition
physical quantity	a numerical magnitude together with a unit
homogeneous equation	an equation in which every term has the same base units
systematic error	an error that shifts every reading in the same direction by the same amount; not reduced by repeating
random error	an error that scatters readings above and below the true value; reduced by repeating and averaging
zero error	the reading an instrument shows when the true value is zero
precision	how close repeated readings are to each other
accuracy	how close a reading, or the mean of several readings, is to the true value
absolute uncertainty	the range within which the true value is expected to lie, in the units of the quantity
percentage uncertainty	the absolute uncertainty divided by the value, multiplied by 100%
scalar	a quantity with magnitude only
vector	a quantity with magnitude and direction

Exam tips

- Give every answer a **unit**, and check **homogeneity** — both sides of an equation must have the same base units.
- Distinguish **random error** (reduce by repeating and averaging) from **systematic error** (a zero or calibration error that repeats do not remove).
- Combine uncertainties: **add absolute** uncertainties when adding/subtracting, **add percentage** uncertainties when multiplying/dividing (and multiply the % by any power).
- Distinguish **precision** (small spread) from **accuracy** (close to the true value).
- Resolve a vector into perpendicular components ($F\cos\theta$, $F\sin\theta$); add vectors tip-to-tail or by components.

Common mistakes

- Substituting 5 cm as 5, or 20 g as 20. Convert the prefix first: 0.05 m, 0.020 kg.
- Giving the percentage uncertainty in T^2 as the same as in T. Squaring doubles it; a square root halves it.
- Treating the percentage uncertainty and the absolute uncertainty as the same number.
- Quoting "the smallest division" as the uncertainty of a reading that was hard to take.
- Offering two answers to a definition. Commit to one.